Hyperchaos and synchronization in two element nonlinear chimney model

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ABSTRACT

The two element chimney model with nonlinearity is studied with the aim of modeling the swaying of trees at high wind speeds. We found solutions for various parameters and also the Lyapunov spectrum numerically. The system is chaotic for a wide range of parameters. We also observed hyperchaos in a subregion of this parameter space. We noticed that the hyperchaos was suppressed when the largest Lyapunov exponent crossed a threshold value. Synchronization between the lower and the upper segments was also studied and, for some parameters, phase synchronization is observed. We also observed transition to antisynchronization and also toggling between the two as the parameters are varied.

The swaying motion of trees has been the subject of several investigations, theoretical as well as experimental, by forest scientists. The reasons for their interest are obviously driven by the losses incurred in stormy conditions. Another place where the understanding of the equations governing the motion of trees is needed is computer animation. The reduced set of equations describing a moving tree will definitely speed up this task, especially when a large number of trees are involved. Though it is evident that the swaying motion of trees is a nonlinear phenomenon, surprisingly, not much effort has been invested in its complete nonlinear analysis. This work is part of such a program to build and understand a complete model step by step.

I. INTRODUCTION

Oscillation of trees driven by wind is always a matter of interest for a broad range of groups, such as forest scientists,[1–3] engineers,[4] animators,[5–7] etc. Their motivations are different, including minimizing damage in stormy wind conditions, improving the stability of the structures, or providing better and more realistic visuals. Trees have a very complex architecture that differs from tree to tree and shows very irregular behavior under high wind and can thus be cited as a natural example of the nonlinear system; but, the nonlinear behavior of this complex system has not been studied much in detail.

The experimental study on the motion of trees has been an active field of research. Several workers have measured various aspects of the movement of trees.[8] They have used various techniques for this purpose, such as using a laser for tracking,[9] measuring the inclination,[10] or using motion tracking from videos,[11] etc.

Theoretically, the earlier attempts for understanding the swaying motion of trees mainly followed two approaches. First, the tree was approximated as a cantilever beam with corresponding partial differential equations describing free vibrations of its central column.[12–14] Second, it was depicted as a chimney model consisting of coupled short oscillating sections with restoring forces at the joints.[15] Both these models have been developed with the assumption that trees are a linear system and neither considered nonlinear restoring forces or even the branched structure.

Since it is known that biological materials exhibit a nonlinear response,[16] the absence of nonlinearity in the models has been particularly striking. Relatively recently, some studies have been initiated that introduce nonlinearity in the restoring force.[17] They considered a basic model consisting of one nonlinear oscillator, and its effect on the resonance was discussed. While some other study[18] introduced a branched structure with nonlinearity in the model, it has not been analyzed much. In order to understand how branching helps in dissipating perturbation and increasing structural stability, Thecke *et al.*[19] studied a Y-shaped branch structure, and Fankem *et al.*[20] carried out a work to test the pagoda system stability, but