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EXISTENCE RESULTS FOR SOLUTIONS TO DISCONTINUOUS DYNAMIC EQUATIONS ON TIME SCALES

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Abstract. In this paper, we present three results about the existence of solutions to discontinuous dynamic equations on time scales. The existence of Carathéodory type solution is produced using convergence and Arzela–Ascoli theorem. The Banach's fixed point theorem is used to investigate the existence and uniqueness of solutions and using Schaefer's fixed point theorem we establish the existence of at least one solution. Our results generalizes and extends some existing theorems in this field.

1. Introduction

The study of dynamic equations on time scales unify and generalize the theory of differential equations and difference equations, it helps to avoid studying results twice. The concept of time scale and dynamic equations on time scales was first introduced by Hilger [12] in his Ph.D. thesis. In the following years, it was realized that dynamic equations on time scales can be applied to hybrid dynamical systems, i.e., in mathematical modelling of any phenomena that involves both continuous and discrete data simultaneously. There have been significant developments and a good deal of research activity devoted to this field. Hence, it become a quite interesting and active research area for researcher across the world. An exhaustive study of dynamic equations on time scales has been done by many authors [1], [4], [6], [7], [14], [15], [16], [25], [26]. In recent years, discontinuous dynamic equations on time scales under various conditions have been studied independently by Gilbert [8], Slavík [23], Satco [20], [21], Santos [18], and Tikare [24].

This paper is concerned with some existence results for discontinuous dynamic equations on an arbitrary finite time scale interval $\mathbb{T}$ such that $\min\mathbb{T}=a$ and $\max\mathbb{T}=b$. We consider the following dynamic problem,

$$\begin{cases} x^{\Delta}(t)=f\big(t,x(t)\big),\ \Delta\text{-a.e. } t\in[a,b)_{\mathbb{T}};\\ x(a)=x_0; \end{cases} \tag{1.1}$$

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