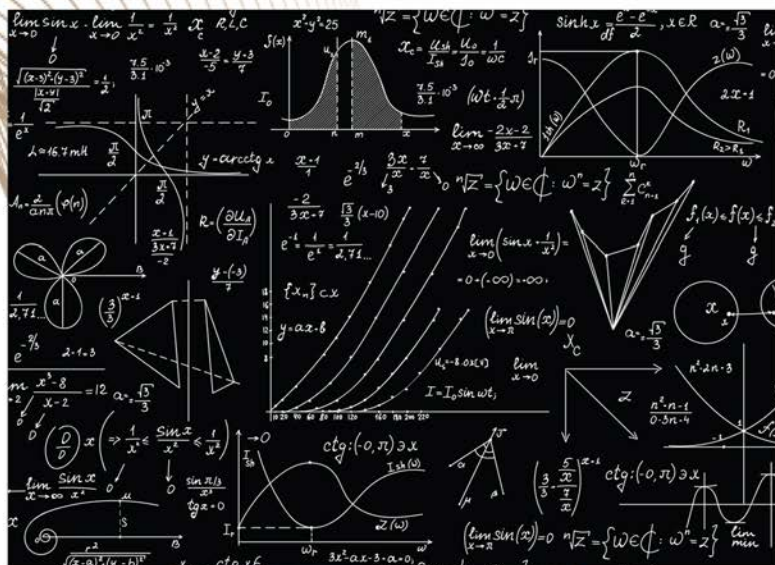


# ADVANCES IN MATHEMATICAL ANALYSIS AND ITS APPLICATIONS



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Edited by  
**BIPAN HAZARIKA**  
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# Advances in Mathematical Analysis and its Applications

*Advances in Mathematical Analysis and its Applications* is designed as a reference text and explores several important aspects of recent developments in the interdisciplinary applications of mathematical analysis (MA) and highlights how MA is now being employed in many areas of scientific research. It discusses theory and problems in real and complex analysis, functional analysis, approximation theory, operator theory, analytic inequalities, the Radon transform, nonlinear analysis, and various applications of interdisciplinary research; some topics are also devoted to specific applications such as the three-body problem, finite element analysis in fluid mechanics, algorithms for difference of monotone operators, a vibrational approach to a financial problem, and more.

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Edited by  
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# Chapter 10

## *Applications of differential transform method on some functional differential equations*

Anil Kumar

Giriraj Methi

Sanket Tikare

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### 10.1 Introduction

The functional differential equations with proportional delay were first studied by Ockendon and Taylor [18] in their work of collecting electric current for the pantograph of an electric locomotive, hence named pantograph equations. The investigation of these equations is important since they find applications in economic activities involving production, planning and decision



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